**Chapter 4 Circuit Theorems**

***Objective and Overview***

The Chapter presents some theorems that apply to electric circuits, and which center primarily around Thevenin’s theorem.

Thevenin’s theorem takes circuit equivalence to its extreme, by representing any LTI circuit, between any two given terminals, by a linear-output voltage source in the case of resistive circuits. Thevenin’s theorem is arguably the most important theorem in circuit analysis, both from theoretical and practical viewpoints. It is therefore discussed at length with several examples that highlight some of its aspects.

The discussion of Thevenin’s equivalent circuit is naturally followed by a discussion of its current-source counterpart, namely, Norton’s equivalent circuit. Norton’s equivalent circuit has the added significance that some circuits may have a Norton’s equivalent circuit, but not a Thevenin’s equivalent circuit, just as the converse is also true.

The Chapter ends with the substitution theorem, which is a useful theorem that simplifies the analysis of some types of circuits, and which can be readily proved using Thevenin’s theorem. A particular form of the substitution theorem, the source absorption theorem, is presented as a useful tool for replacing dependent sources by resistors in some cases, which again simplifies circuit analysis in these cases.

Discussion in this Chapter and in Chapters 5 and 6 is restricted to the dc state.

**4.1 Excitation by Dependent Sources**

Before presenting Thevenin’s theorem, the following concept is discussed:

**Concept: *Dependent sources alone do not excite a stable circuit.***

To illustrate this concept, consider the circuit of Figure 4.1.1. From KCL at the upper essential node, *iY* + 2*IY* = *vX*/6, or,



*vX* = 18*iY* (4.1.1)

From KVL around the mesh on the left, 3*vX* – 4*iY* – *vX* = 0, or,

*vX* = 2*iY* (4.1.2)

Substituting for *iY* from Equation 4.1.2 in Equation 4.1.1,

*vX* = 9*vX*,  or, 8*vX* = 0 (4.1.3)

It follows that *vX* = 0, which also makes *iY* = 0.

Although demonstrated for a particular circuit, it is true in general that if dependent sources are the only sources in a circuit, the circuit is relaxed, that is, all voltages and currents in the circuit are zero. An exception is encountered in circuits with a strong enough positive feedback. These circuits are unstable in that their responses are unbounded, that is, their magnitudes increase with time until they are limited by some nonlinearity in the circuit. Such circuits are not of interest to us in this book. That is the reason for the ‘stable’ qualifier in the statement of the concept enunciated at the beginning of this section.

Although they alone do not excite a circuit, dependent sources do, of course, affect the voltages and currents in the circuit. They can deliver or absorb energy, just like independent sources. It will be shown later (Section 5.1, Chapter 5) that dependent sources effectively modify the values of some resistances in the circuit.

**4.2Thevenin’s Theorem**

In the context of resistive circuits, Thevenin’s theorem can be stated as follows:

**Statement: *A circuit consisting of ideal resistors and sources is equivalent, at a specified pair of terminals, to a linear-output voltage source.***

The equivalent circuit, consisting of an ideal voltage source in series with a resistor is known as Thevenin’s equivalent circuit (TEC). The open-circuit voltage is referred to as the **Thevenin voltage**, *VTh*, and the source resistance as the **Thevenin resistance**, *RTh*. The open-circuit voltage and source resistance were defined for a linear-output voltage source in Section 3.6, Chapter 3.

Thevenin’s theorem takes circuit equivalence to the extreme, in that it reduces any LTI circuit at a given pair of terminals to the simplest possible n equivalent, namely an ideal voltage source in series with a resistor.

Thevenin’s theorem can be illustrated by the simple voltage divider circuit of Figure 4.2.1a supplying a load *RL*. We will derive the relation between *VL* and *IL* at the load terminals ‘ab’.

The circuit is a two-essential-node circuit that can be analyzed using KCL. The current leaving node ‘a’ is:

 (4.2.1)

Equation 4.2.1 can be rearranged as:

 (4.2.2)

Now let us replace the voltage divider by an ideal voltage source *VTh* in series

with an ideal resistor *RTh* (Figure 4.2.1b). KVL gives:



 (4.2.3)

Equation 4.2.3 is of the same form as Equation 4.2.2,

in accordance with Thevenin’s theorem. Moreover, the two equations become identical if *VTh* and *RTh* are given by:

 and  (4.2.4)

Under these conditions, *VTh* in series with *RTh* is equivalent to the voltage divider circuit at terminals ‘ab’, since they have the same *VL*-*IL* relation (Figure 4.2.1c). *RTh* is the source resistance, and *VTh* is the open-circuit voltage at terminals ‘ab’, when *IL* = 0.

Although shown to apply for the voltage divider circuit, Thevenin’s theorem in fact applies to a circuit of any complexity consisting of ideal sources and resistors. Evidently, replacing a complex circuit by its TEC between a given pair of terminals greatly simplifies analysis of the overall circuit, as will be demonstrated on many occasions.

***Derivation of TEC***

In the preceding discussion *VTh* and *RTh* were determined by deriving the *VL*-*IL* relation at the given pair of terminals, which is unnecessarily complicated. A simpler procedure is suggested by the nature of TEC itself and the *VL*-*IL* relation. Since the plot of *VL* vs. *IL* is a straight line (Figure 4.2.1c), this line is uniquely determined by specifying its slope, whose magnitude is *Rsrc*, and a point through which the line passes, such as the intercept on the voltage axis. This intercept is *VTh* and equals *VL*, when *IL* = 0. It can, therefore, be determined directly from the circuit as the open-circuit voltage at terminals ‘ab’. Thus, if *RL* is removed from the circuit (Figure 4.2.2a), it follows from voltage division that:

 (4.2.5)

which is the same as Equation 4.2.4.

*RTh* can be determined in one of two ways: either directly, or as the ratio of the voltage intercept *VTh* in Figure 4.2.1c to the current intercept, which is the short-circuit current *ISC*. When terminals ‘ab’ are short-circuited (Figure 4.2.2b), it follows from Ohm’s law that:



 (4.2.6)

The ratio *VTh*/*ISC* is, from Equations 4.2.5 and 4.2.6, *R*1*R*2/(*R*1 + *R*2), the same as the expression for *RTh* in Equation 4.2.4.



The direct method of determining *RTh* is suggested by Figure 4.2.1b. If *VTh* is set to zero, that is, replaced by a short-circuit (Section 2.4, Chapter 2) and a test voltage source *VT* is applied between terminals ‘ab’, then *RTh* = *VT*/*IT* (Figure 4.2.3a). That is, *RTh* is *Req* seen by the test source between terminals ‘ab’. Alternatively, a test current source *IT* may be applied and *RTh* is again given by *VT*/*IT* (Figure 4.2.3b). But what does setting *VTh* to zero imply in the original circuit? It implies *setting all* *independent sources to zero*, for this removes excitation from the circuit and reduces all currents and voltages in the circuit to zero, including *VTh*. As mentioned in Section 4.1, dependent sources alone do not excite the circuit. Hence, *VTh* becomes zero when all independent sources are set to zero while leaving dependent sources unchanged. Note that although dependent sources alone do not excite the circuit, they do affect the relations between voltages and currents in the circuit in the presence of independent sources by effectively altering the values of resistances, including *RTh* *.* Setting dependent sources to zero will therefore alter *RTh*, whereas setting independent sources to zero removes excitation from the circuit, without

altering *RTh*. Bear in mind that an ideal voltage source is set to zero by replacing it with a short circuit (Section 2.4, Chapter 2), and that an ideal current source is set to zero by replacing it with an open circuit (Section 2.5, Chapter 2),

It should be noted that applying a test source to determine *RTh*, as in Figures 4.2.3a and b, is a formal and general method of determining *Req* between any two terminals of a circuit, with independent sources set to zero. That was, in fact, the method used for determining *Reqs* and *Reqp* in Chapter 3 (Figures 3.2.1 and 3.2.3). Using a test source is the *only* applicable method for determining *Req* in the presence of dependent sources. However, *In the absence of dependent sources*, *Req*, and hence *RTh*, can be determined more directly using series/parallel combinations of resistors, star-delta transformation, etc.

The procedure for deriving TEC can be summarized as follows:

1. *Determine VTh as the open-circuit voltage at the specified terminals*.

2. *Determine the short-circuit current ISC at the specified terminals, which gives RTh as VTh*/*ISC*.

3. *Set all independent sources in the given circuit to zero, leaving dependent sources unchanged. RTh is the resistance Req looking into the specified terminals. Formally, this resistance is obtained by applying a test voltage source or a test current source and determining Req as the ratio of the voltage at the source terminals to the source current. In the absence of dependent sources, Req can be derived directly from series/parallel combinations of resistors, and using star-delta transformations, if necessary*.

The following should be noted concerning this procedure:

1. Since *VTh = RThISC*, only two of the three quantities in this relation need be determined through the aforementioned three steps. However, it is useful for checking purposes to determine all three of these quantities independently.
2. Moreover, some of the aforementioned three steps many be easier to implement than others. Thus, setting independent sources to zero can make the circuit particularly simple.
3. In some cases, *VTh* = 0, which means that *ISC* = 0. It follows that *VTh*/*ISC* is 0/0, which is indeterminate. In this case, *RTh* can only be determined by Step 3 of the aforementioned procedure. This is illustrated by Example 4.2.3.
4. A potential ambiguity in deriving TEC at a pair of terminals is whether or not to include in TEC a resistor *R* that is connected between the given pair of terminals. The ambiguity is resolved in this book by the way the resistor is drawn with respect to the given terminals or by the way TEC is required. In Figure 4.2.4a, for example, *R* is drawn beyond the terminals at which TEC is required, as was done

in Figure 4.2.1a. The implication is that *R* should *not* be included in TEC. Even without drawing the terminals in this manner, requiring “TEC seen by *R*”, or “TEC looking into terminals ‘ab’” means that *R* should not be included in TEC. On the other hand, if the given terminals are beyond ‘*R’*, as in Figure 4.2.4b, the implication is that *R* should be included in TEC. Requiring “TEC between terminals ‘ab’” is unambiguous in this case.



1. Even in the case of Figure 4.2.4b, it may be advantageous to remove *R* and determine an intermediate TEC from the simpler circuit that results when *R* is removed. After obtaining this TEC, *R* is connected to the terminals of this TEC and the final TEC derived. This procedure is illustrated by Example 4.2.4.

**🞺*Derivation of TEC with PSpice***

Although *VTh*, *ISC*, and *RTh* can be derived from two separate simulations, it is possible, and more convenient, to derive TEC from a single simulation. The basis for this procedure is instructive, and can be explained with reference to Figure 4.2.5a. A test current source *IT* is applied at terminals ‘ab’, between which TEC is to be derived. This TEC, consisting of *VTh* and *RTh* is shown between these terminals, which signifies that the original circuit is left as is, that is, with the *independent sources retained*, so that *VTh* ≠ 0. KVL gives:



 (4.2.8)

If *IT* is varied between 0 and 1 A, and *VT* is plotted against *IT*, a straight line graph is obtained having a voltage intercept *VTh* at *IT* = 0. Let *VT*1 denote *VT* at *IT* = 1 A. From equation 4.2.8, the difference (*VT*1 – *VTh*) is numerically equal to *RTh* when *IT* = 1 A (Figure 4.2.5b). *IT* is conveniently varied in PSpice over a desired range of values, using the “DC Sweep” feature, as explained in Example 4.2.1.

**Example 4.2.1**

It is required to determine *IL* in Figure 4.2.6 by deriving TEC looking into terminals ‘ab’.

**Solution:** When terminals ‘ab’ are open circuited by removing the 50 Ω resistor, the circuit becomes as in Figure 4.2.7a. With no current in the 20 Ω resistor, the 6 A source current flows through the 30 Ω resistor, producing a voltage of 180 V across this resistor. From KVL, *VTh* = 180 + 20 = 200V.



When terminals ‘ab’ are short circuited, the short-circuit current *ISC* can be determined by transforming the ideal 6 A source in parallel with 30 Ω to an ideal voltage source of 30×6 = 180 V in series with 30 Ω, as in Figure 4.2.7b. *ISC* now flows through the 30 Ω and 20 Ω resistors. Applying KVL, starting from node ‘b’ and going clockwise: 180 – 30*ISC* – 20*ISC* + 20 = 0. This gives *ISC* = 200/50 = 4 A. It follows that *RTh* = *VTh*/*ISC* = 200/4 = 50 Ω.

Since the circuit does not have dependent sources, it is not necessary to apply a test source and determine the ratio of the voltage of the test source to the current through the source. *RTh* can be determined in this case as *Req* between terminals ‘ab’, or *Rin*, the input resistance looking into terminals ‘ab’, with independent sources set to zero. The 20 V source is replaced by a short circuit and the 6 A source is replaced by an open circuit, as in Figure 4.2.8a.



The resistance looking into terminals ‘ab’ is seen to be 30 + 20 = 50 Ω, as determined previously. TEC between terminals ‘ab’ is therefore a 200 V source in series with 50 Ω (Figure 4.2.8b). When the 50 Ω resistor is connected between terminals ‘ab’ *IL* is given by: *IL* = 200/(50 + 50) = 2 A.



**Simulation:** The circuit is entered as in Figure 4.2.9. An IDC I2 is connected between terminals ‘ab’ of the circuit. Its default value of 0A need not be changed. A voltage marker is placed at terminal ‘a’ of the circuit. In the Simulation Settings, ‘Analysis type’ is ‘DC Sweep’, ‘Primary Sweep’ is selected under ‘Options’, ‘Current source’ is selected as ‘Sweep variable’, and I2 is entered in the ‘Name’ field. ‘Sweep type’ is ‘Linear’, ‘Start value’ is 0, ‘End value’ is 1, and ‘Increment’ is 1m, which is small enough to give a large number of points (1000) and hence a smooth line. When the simulation is run, Figure 4.2.10 is displayed. Cursor 1 is positioned at I2 = 1 A and cursor 2 at I2 = 0. In the cursor window, *VTh* is read as Y2 = 200.000, and *RTh*×1 is read as Y2 – Y1 = 50.000.



**Exercise 4.2.1**

Verify that applying a test voltage source or a test current source in Figure 4.2.8a gives the same *RTh.*

**Example 4.2.2**

It is required to derive TEC seen by the 26 Ω load in Figure 4.2.11.

**Solution:** Terminals ‘ab’ are open circuited by removing the 26 Ω load, so that *VTh* is now the voltage between these terminals (Figure 4.2.12). The 2 A current now flows in the 8 Ω resistor between nodes ‘a’ and ‘d’, and *Ib* flows through the 4 V source and the 4 Ω resistor. The current leaving node ‘c’, through the VCCS is, from KCL, (*Ib* – 2). Node ‘d’ can then be used to check that KCL is satisfied in the circuit. The current entering node ‘d’ is (*Ib* – 2 + 2) = *Ib*, the same as the current leaving the node. As emphasized previously, it is good practice in problem-solving not to use additional variables and to mark the currents and voltages on the circuit diagram.



*Ib* is determined from KVL around mesh ‘bcdb’. By going clockwise around this mesh, starting from node ‘b’, KVL gives: 4 – 4*Ib* – 4*Ib* – 8*Ib* = 0, so that *Ib* = 4/16 = 0.25 A, and *VTh* = 8×2 + 8×0.25 = 18 V. Note that there is no point in taking KVL around any mesh that includes the 2 A source because the voltage across this source is an additional unknown.



When terminals ‘ab’ are short-circuited, the circuit becomes as in Figure 4.2.13. Because *Vda* = *Vdb*, and the resistances in the branches ‘da’ and ‘db’ are equal, it follows that the current *Ida* is

also *Ib*. From KCL at node ‘a’, the current in the short circuit is *ISC* = (2 + *Ib*). From KCL at node ‘b’, the current in the branch ‘bc’ is (2 + 2*Ib*), and from KCL at node ‘c’, the current in the VCVS is 2*Ib*. Again, KCL at node ‘d’ can serve as a check on KCL in the circuit.

By going clockwise around the mesh ‘bcdb’, KVL gives: 4 – 4(2 + 2*Ib*) – 4*Ib* – 8*Ib* = 0. Hence, *Ib* = -4/20*Ib* = -0.2 A, so that *ISC* = (2 + *Ib*) = 1.8 A. It follows that *RTh* = 18/1.8 = 10 Ω. It should be noted that *the assigned positive direction of ISC should be consistent with that of VTh. Otherwise, the sign of RTh will be incorrect.* The positive direction of *ISC* is that of the voltage drop *VTh* at the open-circuited terminals. This is the direction of flow of current at the output terminals under the influence of a voltage drop between these terminals.



To determine *RTh* by applying a test source, the independent sources are set to zero, so that the 4 V source is replaced by a short circuit and the 2 A source by an open circuit, as shown in Figure 4.2.14. With a 1 A test source applied, KCL

is satisfied at node ‘d’ by having a current (1 – *Ib*) leaving this node through the VCVS and the 4 Ω resistor. Node ‘b’ can be used to check KCL. By going clockwise around the mesh ‘bcdb’, KVL gives: 4(1 – *Ib*) – 4*Ib* – 8*Ib* = 0. Hence, *Ib* = 4/16 = 0.25 A, and *VT* = 8×1 + 8×0.25 = 10 V. It follows that *RTh* = (10 V)/(1 A) = 10 Ω, as before.

**Simulation:** The circuit is entered as in Figure 4.2.15. Proceeding as in Example 4.2.1, Figure 4.2.16 is displayed when the simulation is run. It is seen that *VTh* = 18 V and *RTh* = 10 Ω.

**Problem-Solving Tip**

* In deriving TEC, the assigned positive direction of *ISC* should be in the direction of the voltage drop *VTh* so as to obtain the correct sign of *RTh* = *VTh*/*ISC*.

**Exercise 4.2.2**

Determine *RTh* in Figure 4.2.14 by applying a 1 V test source rather than a 1 A source.

**Example 4.2.3**

It is required to derive TEC seen by the 25 Ω resistor in Figure 4.2.17a.

**Solution:** As mentioned previously, the circuit configuration is a bridge circuit, since the 25 Ω resistor between nodes ‘c’ and ‘d’ is a “crossover” element, like a bridge. To derive TEC seen by the 25 Ω resistor, the resistor is removed and *Vbc* determined (Figure 4.2.17b). The voltages *Vbd* and *Vcd* in Figure 4.2.17b can be determined from



voltage division, since the 10 Ω resistor is in series with the 15 Ω resistor, and the 20 Ω resistor is in series with the 30 Ω resistor. Hence, *Vbd* = 5×30/(20 + 30) = 3 V, and *Vcd* = 5×15/(10 + 15) = 3 V.

When the two middle nodes of the bridge circuit are at the same voltage, that is, *Vbd* = *Vcd*, the bridge is said to be “balanced”, as will be discussed more fully in Appendix 5A, Chapter 5.

With the bridge balanced, *VTh* = *Vbc* = *Vbd* – *Vcd* = 0. Moreover, when nodes ‘b’ and ‘c’ are at the same voltage, then the current through any resistor connected between these nodes is zero, because there is no voltage to drive such a current. The current remains zero as the resistance is reduced to zero, that is, when nodes ‘b’ and ‘c’ are short circuited. It follows that *ISC* = 0. However, having *VTh* and *ISC* equal to zero *does not* mean that *RTh* = 0, because *RTh* = *VTh*/*ISC* is indeterminate and could be finite.



To determine *RTh*, therefore, the resistance looking into terminals ‘bc’ should be derived, with the independent voltage source set to zero, that is, replaced by a short-circuit (Figure 4.2.18a). To make it easier to visualize the connections, it is helpful to redraw the circuit as in Figure 4.2.18b. Clearly, the resistance between nodes ‘c’ and ‘d’ is (10||15 + 20||30) = 18 Ω. With *VTh* = 0, TEC reduces to an 18 Ω resistor.



**Simulation:** The circuit is entered as in Figure 4.2.19. Proceeding as in Example

4.2.1, Figure 4.2.20 is displayed when the simulation is run. It is seen that *VTh* = 0 and *RTh* = 18 Ω.

**Example 4.2.4**

It is required to determine *IX* in Figure 4.2.21 using TEC.



**Solution:** TEC will be derived in two steps. The first step is to determine TEC as seen by the 3 A source in parallel with the 6 Ω resistor, as illustrated in Figure 4.2.22a. The motivation for this step is that the parallel combination of the 3 A source and the 6 Ω resistor is connected between the terminals where TEC is required. Under these conditions this parallel combination can be temporarily removed, resulting in a considerably simpler circuit for which an intermediate TEC can be more easily derived. The 3 A source is then be added to this intermediate TEC and a new TEC derived, from which *IX* is determined by adding the 6 Ω resistor.



It is seen from voltage division in Figure 4.2.22a that *Vac* = 9×6/9 = 6 V, and *Vbc* = 9×3/9 = 3 V. It follows that *VTh*1 = *Vac* – *Vbc* = 6 – 3 = 3 V. *RTh*1 is most easily

found by determining the resistance between terminals ‘ab’ with the 9 V source set to zero, that is, replaced by a short circuit. This makes *RTh*1 = *Rab* = (6||3) + (6||3) = 2 + 2 = 4 Ω. The intermediate TEC will therefore consist of *VTh*1 = 3 V is series with *RTh*1 = 4 Ω (Figure 4.2.22b).

The next step is to connect the 3 A current source and derive a second TEC as seen by the 6 Ω resistor (Figure 4.2.23a). The 3A current now flows through the 3 V source and the 4 Ω resistor, so that the open-circuit voltage *Vab* = *VTh*2 = 3 + 12 =

15 V. When the 3 A source is replaced by an open circuit and the 3 V source by a short circuit, the resistance seen between terminals ‘ab’ is *RTh*2 = *Rab* = 4 Ω as before (Figure 4.2.23b). When the 6 Ω resistor is connected to terminals ‘ab’, *IX* that flows is 15/(6 + 4) = 1.5 A (Figure 4.2.23c).



**Simulation:** Although the current *IX* can be derived directly by simulating the circuit of Figure 4.2.21 without invoking TEC, it is instructive to derive by simulation TEC as seen by the 6 Ω resistor. The circuit is entered as in Figure 4.2.24. Proceeding as in Example 4.2.1, Figure 4.2.21 is displayed when the simulation is run. It is seen that *VTh* = 15 V and *RTh* = 4 Ω, as determined previously.



**Problem-Solving Tips**

* When a voltage or a current in a circuit is required, it can often be conveniently determined by deriving TEC at the terminals associated with this voltage or current.
* In deriving TEC it is often advantageous to temporarily remove elements that appear in parallel or in series with the terminals between which TEC is required,

derive an intermediate TEC, then restore the removed elements to this intermediate TEC in order to derive the final TEC.

**Exercise 4.2.3**

Determine *RTh* in Figure 4.2.22a by deriving *ISC*.

**Primal Exercise 4.2.4**



Determine TEC between nodes ‘a’ and ‘b’ in Figure 4.2.26: (a) without including the 60 Ω resistance between these nodes; (b) including this resistance.

Ans. (a) 12 V in series with 30 Ω;

(b) 8 V in series with 20 Ω.

**4.3Norton’s Theorem**

In the context of resistive circuits, Norton’s theorem can be stated as follows:

**Statement: *A circuit consisting of ideal resistors and sources is equivalent, at a specified pair of terminals, to a linear-output current source.***

It is seen that **Norton’s equivalent circuit** (NEC) is in fact the linear-



output current source equivalent of TEC. The two equivalent circuits are related by source transformation, as illustrated in Figure 4.3.1. The ideal current source *IN* is referred to as Norton’s current and is the short-circuit current of TEC, that is, *VTh*/*RTh*, in accordance with source transformation. The source resistance that is in parallel with *IN* is Norton’s resistance *RN*, and is the same as *RTh* in TEC.

It is sometimes more convenient to derive NEC rather than TEC, as in Example 4.3.1. Moreover, some circuits may have an NEC but not a TEC, or conversely, as in the case of ideal voltage sources and ideal current sources. Thus, an ideal voltage source can be considered to be its own TEC, with *RTh* = 0. *IN* = *VTh*/0

→ ∞, which means that NEC does not exist. Similarly, an ideal current source can be regarded as its own NEC, with *RN* = ∞, so that *VTh* → ∞, which means that TEC does not exist. This is in accordance with the fact that an ideal voltage source cannot be transformed to an ideal current source, and conversely, as explained in Section 3.6, Chapter 3. Circuits that have TEC but not NEC generally reduce to an ideal voltage source between the terminals involved, whereas circuits that have NEC but not TEC generally reduce to an ideal current source between these terminals. Examples of these are given in the problems at the end of the chapter.

The procedure for deriving NEC is essentially the same as that for TEC. When independent sources are set to zero in Figure 4.3.1, the ideal voltage source in TEC is replaced by a short circuit and the ideal current source in NEC is replaced by an open circuit. The resistance looking into terminals ‘ab’ is *RTh* = *RN* in both cases. In the case of TEC, *VTh* is generally determined directly, and the short-circuit current, *IN*, is determined as an alternative method for finding *RTh*. In the case of NEC, *IN* is generally determined directly, and the open-circuit voltage, *VTh*, is determined as an alternative method for finding *RN*.

**🞺*Derivation of NEC with PSpice***

NEC can be derived in a single simulation, analogous to that described for TEC, and explained in Figure 4.3.2a. A test source *VT* is connected between these terminals with the circuit left as is, that is with the independent sources retained, so that *IN* ≠ 0.



KCL gives:

 (4.3.1)

If *VT* is varied between 0 and 1 V, and *IT* is plotted against *VT*, a straight line graph is obtained having a current intercept *IN* at *VT* = 0. The difference between *IT*1, which is *IT* at *VT* = 1 V, and *IN* is numerically equal to *GN* (Figure 4.3.2b). *VT* is conveniently varied in PSpice over a desired range of values, using the “DC Sweep” feature.

**Example 4.3.1**

It is required to determine *IL* in Figure 4.3.3 using NEC.



**Solution:** When terminals ‘ab’ are short circuited, *VX* = 0 and the VCVS becomes a short circuit (Figure 4.3.4a). The 15 Ω and 10 Ω resistors are paralleled. By current division, the currents in these two resistors are 2 A and 3 A as shown. *IS* = -2 A and the CCCS becomes 6 A directed upwards. It follows that *ISC* = *IN* = 3 + 6 = 9 A. Note that the 6 A source current only adds to the current in the short circuit between nodes ‘a’ and ‘b’ in Figure 4.3.4a and does not affect current division.



To determine *RN*, a test source *IT* is applied, with the 5 A current source replaced by an open circuit (Figure 4.3.4b). KVL gives:

2*VT* – 25*IS* –*VT* = 0, or *VT* = 25*IS* (4.3.2)

From KCL, *IS* + *IT* = 3*IS*, or,

*IT* = 2*IS* (4.3.3)

Dividing Equation 4.3.3 by Equation 4.3.2,

Ω (4.3.4)

NEC therefore consists of a 9 A source in parallel with a 12.5 Ω resistor (Figure 4.3.5a). When the 50 Ω resistor is connected between terminals ‘ab’, it follows from current division that:

 = 1.8 A (4.3.5)

It may be noted that determining the open-circuit voltage in Figure 4.3.3 in order to work with TEC is slightly more complicated than determining *IN* as in Figure 4.3.4a (Exercise 4.3.1).



**Simulation:** The circuit is entered as in In Figure 4.3.6, in accordance with the method explained in connection with Figure 4.3.2. The 50 Ω resistor is included in order to facilitate finding *IL*, as explained



later, but could be left out for the purpose of determining NEC between terminals ‘ab’. Note the alternative way of connecting dependent sources in Figure 4.3.6 in order to avoid making the somewhat awkward connections to the control terminals of dependent sources. This is to label the appropriate nodes using the net alias feature of PSpice, as described in Appendix A3. PSpice considers nodes having the same label to be connected together, as shown in Figure 4.3.6. A DC sweep is performed as described in Example 4.2.1 but sweeping a voltage source instead of a current source. The DC sweep gives the plot of Figure 4.3.7, from which, *IN* = 9 A and *GN* = 1/0.1 = 10 Ω, this being the parallel resistance of 12.5 Ω and 50 Ω (Figure 4.3.5b). *IL*

is determined from *Vab* = 9×10 = 90 V in both Figures 4.3.5a and 4.3.5b. It follows from Figure 4.3.5a that *IL* = 90/50 = 1.8 A.

**Exercise 4.3.1**

Determine *VTh* directly from the circuit of Figure 4.3.3.

**Primal Exercise 4.3.2**



Determine NEC between nodes ‘a’ and ‘b’ in Figure 4.3.8: (a) without including the 10 Ω resistance between these nodes; (b) including this resistance.

Ans. (a) 10 A in parallel with 15 Ω;

(b) 10 A in parallel with 6 Ω.

**4.4 Substitution Theorem**

Consider a circuit N connected at terminals ‘ab’ to a circuit NA having a designated voltage *VA* across ‘ab’ (Figure 4.4.1a), where *VA* could be of known or unknown value. Let *IX* be the current flowing from N to NA. According to the substitution theorem, NA can be replaced by an independent voltage source *VA* (Figure 4.4.1b), without affecting *IX*. This can be readily justified if N is represented between terminals ‘ab’ by its TEC, as in Figures 4.4.1c and 4.4.1d. It is evident from these figures that KVL is the same in both cases, namely:  and gives:



 (4.4.1)

In other words, replacing NA by an independent voltage source *VA* does not affect N, since *IX* remains the same.

Similarly, suppose that NA has a designated current *IA* at the common terminals ‘ab’ (Figure 4.4.2a), where *IA* could be known or unknown. Let *VX* be the voltage across terminals ‘ab’. According to the substitution theorem, an independent current source *IA* can be substituted for NA (Figure 4.4.2b), without affecting *VX*. Again, this can be justified if N is represented between terminals ‘ab’ by its TEC, as in Figures 4.4.2c and 4.4.2d. It is evident from these figures that KVL is the same in both cases and gives:



 (4.4.2)

In other words, replacing NA by an independent current source *IA* does not affect N, since *VX* remains the same. The substitution theorem can be stated as follows:

**Statement: *A circuit having a designated voltage V across it can be replaced by an ideal, independent voltage source V, without affecting the rest of the circuit. Similarly, a circuit having a designated current I through it can be replaced by an ideal, independent current source I, without affecting the rest of the circuit.***

There is no restriction on the nature of the circuit NA that is being replaced by an independent source. It could be a single resistor, a dependent source, or any valid combination of independent sources, dependent sources, and resistors. The substitution theorem is illustrated by Example 4.4.1; it is particularly useful in connection with superposition, discussed in the following chapter.

**Exercise 4.4.1**

Justify the substitution theorem by replacing circuit N by its NEC: (a) in Figure 4.4.1; and (b) in Figure 4.4.2.

**Example 4.4.1**

Given a known bridge circuit connected to a circuit NA of unknown component values, as illustrated in Figure 4.4.3. The bridge circuit is inaccessible for measurements, but the voltage across NA can be measured by means of a voltage-measuring device (a voltmeter) and is found to be 15 V, of the polarity indicated. It is required to determine *IS*, the current drain on the 6 V battery.



**Solution:** This may look like an impossible problem, but *IS* can be readily determined by means of the substitution theorem. According to this theorem, NA can be replaced by a 15 V independent source, without disturbing the circuit (Figure 4.4.4a). *IS* can be conveniently determined by deriving TEC between terminals ‘ad’ (Figure 4.4.4b). *VTh* = *Vad* = *Vab* + *Vbd*. From voltage division, *Vab* =  -9 V, and *Vbd* =



 10 V. It follows that *VTh* = *Vad* = -9 + 10 = 1 V.

*RTh* is determined as *Req* between terminals ‘a’ and ’d’ with the 15 V source set to zero (Figure 4.4.5a), which makes nodes ‘b’ and ‘c’ one and the same. The resistance between terminals ‘a’ and ‘b’ is 15||10  Ω. The resistance between terminals ‘c’ and ‘d’ is 30||15  Ω. Hence, *RTh* = 10 + 6 = 16 Ω. Replacing the load circuit between terminals ‘a’ and ’d’ by its TEC, the circuit becomes as shown in Figure 4.4.5b. It follows from KVL that (4 + 1) = (4 + 16)*IS*, which gives *IS* = 5/20 = 0.25 A.



**Simulation:** The circuit is entered in as in Figure 4.4.6. After selecting ‘Bias Point’ in the simulation profile and running the simulation, pressing the I and V buttons displays the currents and voltages indicated in Figure 4.4.6. It is seen that *Vad* = 5 V and *IS* = 0.25 A.

**Exercise 4.4.2**

Determine *RTh* in Example 4.4.1 by applying: (a) a 1 A test source; (b) a 1 V test source.

**Primal Exercise 4.4.3**



Consider the circuit of Figure 4.4.7. Determine: (a) the independent voltage source that can replace the 5 Ω resistor without affecting the current *I* in the circuit; (b) the independent current source that can replace this tresistor without affecting *Vab*.

Ans. (a) 5 V, with node ‘a’ positive with respect to ‘b’; (b) 1 A directed from node ‘a’

to ‘b’.

**4.5 Source Absorption Theorem**

The source absorption theorem is a special case of the substitution theorem that can be usefully applied in some cases involving dependent sources, particularly in transistor circuits. In the definition of dependent sources (Section 2.6, Chapter 2), it was stated that the controlling variable is a current or voltage elsewhere in the circuit,

which excludes the controlling variable being that of the source itself, or a quantity proportional to it. In these cases, the dependent source can be conveniently replaced by a resistor.

**Concept: *If a direct proportionality exists between the voltage across a dependent source and the source current, the dependent source can be replaced by a resistor having a resistance equal to the ratio of the voltage across the source to the source current.***

To justify this, consider the dependent voltage source of Figure 4.5.1a, where the source voltage is proportional to the current through the source, *V* = *ρI*. If the dependent source is replaced by a resistor having *R* = *ρI/I = ρ*, then for the same *I* through the two circuit elements, the voltage *V* across them is the same. The dependent voltage source having *V* = *ρI* is therefore equivalent to a resistor *R* and can be replaced by this resistor between the same terminals.



The dependent current source of Figure 4.5.1b has *I* = *σV*, where *V* is the voltage across the source. The source can be replaced by a resistor having *R* = *V/σV* = 1/*σ*. The two circuit elements are equivalent since, for the same voltage across them, the current through them is the same.

Note that in both Figures 4.5.1a and 4.5.1b, a positive value of *R* corresponds to having current in the dependent source in the direction of a voltage drop across the source.

**Example 4.5.1**

A case that is often encountered in transistor circuits is that of Figure 4.5.2a, where *Req* between terminals ‘ab’ is required.

**Solution:** The current source *gmvx* is transformed to a voltage source *gmvxro* in series with *ro* (Figure 4.5.2b). The current *i* through *Rx* is *vx* /*Rx*, which makes the source voltage *gmvxro* proportional to the current *vx* /*Rx* through the source. The dependent source can therefore be replaced by a resistance whose value is the source voltage divided by the source current. This resistance is (*gmvxro*)/(*vx* /*Rx*) = *gmroRx*; *Req*.

between terminals ‘ab’ is then the sum of the three resistances in Figure 4.5.2c:



*Req* = *ro* + *gmroRx +* *Rx*

(4.5.1)

**Exercise 4.5.1**

Derive *Req* by applying: (a) a 1 A test source; (b) a 1 V test source.



**Primal Exercise 4.5.2**

Determine *Rin* in Figure 4.5.3 using the source absorption theorem based on the current that flows through the source.

Ans. 40 Ω.

**Summary of Main Concepts and Results**

* Dependent sources alone do not excite a stable circuit. They affect currents and voltages in the circuit by effectively modifying the values of some resistances in the circuit.
* Thevenin’s Theorem: A circuit consisting of ideal resistors and sources is equivalent, at a specified pair of terminals, to a linear-output voltage source. The voltage of the ideal voltage source element is referred to as the Thevenin voltage, *VTh*, and the source resistance as the Thevenin resistance, *RTh*.
* *VTh* is determined as the open-circuit voltage at the specified terminals. *RTh* can be determined as *VTh*/*ISC*, where *ISC* is the short-circuit current between the specified terminals.
* *RTh* can also be determined by setting independent sources in the circuit to zero and applying a test voltage source or a test current source. *RTh* is then the ratio of the voltage at the source terminals to the source current. In the absence of dependent sources this resistance can be derived directly from series/parallel combinations of resistors, and using star-delta transformations, if necessary.
* Norton’s Theorem: A circuit consisting of ideal resistors and sources is equivalent, at a specified pair of terminals, to a linear-output current source.
* NEC follows from TEC through source transformation.
* According to the substitution theorem, a circuit having a designated voltage *V* across it can be replaced by an ideal, independent voltage source *V*, without affecting the rest of the circuit. Similarly, a circuit having a designated current *I* through it can be replaced by an ideal, independent current source *I*, without affecting the rest of the circuit. The circuit of unknown component values could be a single resistor, a dependent source, or any valid combination of independent sources, dependent sources, and resistors.
* If a direct proportionality exists between the voltage across a dependent source and the source current, the dependent source can be replaced by a resistor having a resistance equal to the ratio of the voltage across the source to the source current. The resistance value is positive when the source current is in the direction of a voltage drop across the source.

**Problem-Solving Tips**

1. In deriving TEC, the assigned positive direction of *ISC* should be in the direction of the voltage drop *VTh* so as to obtain the correct sign of *RTh* = *VTh*/*ISC*.
2. When a voltage or a current in a circuit is required, it can often be conveniently determined by deriving TEC at the terminals associated with this voltage or current.
3. In deriving TEC it is often advantageous to temporarily remove elements that appear in parallel or in series with the terminals between which TEC is required, derive an intermediate TEC, then restore the removed elements to this intermediate TEC in order to derive the final TEC.

**Exercises and Problems**

***Apply IDSEPIC and verify solutions by PSpice simulation whenever feasible.***

**P4.1 TEC and NEC**

**P4.1.1** Determine *IX* in Figure P4.1.1 in two ways: (a) by deriving TEC for each half-circuit and combining the two TECs; (b) By deriving a single TEC between the two terminals through which *IX* flows.



Ans. 0.75 A.

**P4.1.2** (a) Determine *VSRC* in Figure P4.1.2 by deriving TEC between terminals ‘bc’. (b) Determine *ISRC*, *VX*, and *VY*.

Ans. (a) 120 V; (b) 12 A, *VX* = 48 V, *VY* = 72 V.



**P4.1.3** Derive TEC looking into terminals ‘ab’ in Figure P4.1.3.

Ans. *VTh* = 4 V, *RTh* = 4 Ω.



**P4.1.4** Derive TEC looking into terminals ‘ab’ in Figure P4.1.4.

Ans. *VTh* = 12 V, *RTh* = 6 Ω.

**P4.1.5** Derive TEC and NEC looking into terminals ‘ab’ in Figure P4.1.5.



Ans. *VTh* = 0 = *IN*, *RTh* = *RN* = 25 Ω.



**P4.1.6** Derive TEC and NEC looking into terminals ‘ab’ in Figure P4.1.6.

Ans. TEC is an ideal 5V source; NEC does not exist.

**P4.1.7** Derive TEC and NEC looking into terminals ‘ab’ in Figure P4.1.7.



Ans. NEC is an ideal 8A source; TEC does not exist.



**P4.1.8** Derive TEC and NEC looking into terminals ab in Figure P4.1.8, assuming: (a) *α* = 1; and (b) *α* = 2.

Ans. (a) *VTh* = 0, *RTh* = 1 Ω; (b) *VTh* = -5 V, *RTh* = 0, NEC does not exist.

**P4.1.9** Derive TEC looking into terminals ‘ab’ in Figure P4.1.9.



Ans. *VTh* = 16 V, *RTh* = 8 Ω.



**P4.1.10** Derive TEC looking into terminals ‘ab’ in Figure P4.1.10.

Ans. *VTh* = 3 V, *RTh* = 75 Ω.



**P4.1.11** Derive NEC looking into terminals ‘ab’ in Figure P4.1.11.

Ans. *IN* = 4.4 A, *GN* = 0.04 S.



**P4.1.12** Derive NEC looking into terminals ‘ab’ in Figure P4.1.12.

Ans. *IN* = 0.3 A, *GN* = 0.025 S.



**P4.1.13** Derive TEC looking into terminals ‘ab’ in Figure P4.1.13.

Ans. *VTh* = 10 V, *RTh* = 10 Ω.

**P4.1.14** Derive TEC looking into terminals ‘ab’ in Figure P4.1.14.



Ans. *VTh* = 0, *RTh* = 25 Ω.



**P4.1.15** Derive TEC looking into terminals ‘ab’ in Figure P4.1.15.

Ans. *VTh* = 40 V, *RTh* = 0.

**P4.1.16** Derive TEC as seen by the 50 Ω resistor in Figure P4.1.16.



Ans. *VTh* = 20 V, *RTh* = 10 Ω.



**P4.1.17** Determine NEC looking into terminals ‘ab’ in Figure P4.1.17.

Ans. *IN* = 0, *RN* = 20/3 Ω.

**P4.1.18** Determine TEC looking into terminals ‘ab’ in Figure P4.1.18, given *R* = 1 Ω.



Ans. *VTh =* 1 V, *RTh =* 0.5 Ω.



**P4.1.19** Determine *VO* in Figure P4.1.19 using TEC.

Ans. 20 V.



**P4.1.20** Determine *IO* in Figure P4.1.20 using NEC.

Ans. 20 A.



**P4.1.21** Determine *VO* in Figure P4.1.21 using TEC.

Ans. 15.51 V.

**P4.1.22** Determine *IO* in Figure P4.1.22 using NEC.



Ans. 15.51 A.



**P4.1.23** Determine *VO* in Figure P4.1.23 using TEC.

Ans. -10/3 V.



**P4.1.24** Determine *IO* in Figure P4.1.24 using NEC.

Ans. -10/3 A.

**P4.1.25** Determine *VO* in Figure P4.1.25 using NEC. Note that circuit does not posses a TEC.



Ans. 30 V.



**P4.1.26** Determine

*IO* in Figure P4.1.26 using TEC. Note that the circuit does not possess an NEC.

Ans. 30 A.



**P4.1.27** Determine TEC looking into terminals ‘ab’ in Figure P4.1.27.

Ans. *VTh* = 27 V, *RTh* = 3 Ω.



**P4.1.28** Determine *RTh* looking into terminals ‘ab’ in Figure P4.1.28.

Ans. *RTh* = 100/3 Ω

**P4.1.29** Derive TEC looking



into terminals ‘ab’

in Figure P4.1.29.

Ans. *VTh* = 12 V, *RTh* = 80 Ω.

**P4.1.30** Derive TEC looking into terminals ‘ab’ in Figure P4.1.30.



Ans. *VTh* = -100/3 V , *RTh* = 1000/3 Ω.



**P4.1.31** Derive TEC looking into terminals ‘ab’ in Figure P4.1.31.

Ans. *VTh* = 4 V, *RTh* = 2 Ω.



**P4.1.32** Determine *IO* in Figure P4.1.32 using TEC.

Ans. -0.65 A.

**P4.1.33** Derive TEC between terminals ‘ab’ in Figure P4.1.33.



Ans. *VTh* = 5 V, *RTh* = 5 Ω

**\*P4.1.34** Derive TEC between terminals ‘ab’ in Figure P4.1.34.



Ans. *VTh* = -1 V, *RTh* = 1.5 kΩ.



**P4.1.35** Derive TEC

between nodes ‘ab’ in Figure P4.1.35.

Ans. *VTh* = 20 V, *RTh* = 8 Ω.



**\*P4.1.36** Derive TEC between terminals ‘ab’ in Figure P4.1.36.

Ans. *VTh* = 80 V, *RTh* = 10 Ω.



**\*P4.1.37** Derive NEC between terminals ‘ab’ in Figure P4.1.37, assuming all resistances are 2 Ω.

Ans. -41/33 A, 66/23 Ω.

**P4.1.38** Determine *R* so that Norton’s current between nodes ‘ab’ in Figure P4.1.38 is zero.



Ans. 1 Ω.



**P4.1.39** Derive NEC looking into terminals ‘ab’ in Figure P4.1.39.

Ans. *IN* = 0.5 A, *RN* = 10 Ω.



**P4.1.40** Derive TEC as seen by *RL* in Figure P4.1.40.

Ans. *VTh* = 2.5 V, 0.5 kΩ.



**\*P4.1.41** Derive TEC looking into terminals ‘ab’ in Figure P4.1.41.

Ans. *VTh* = 1 V, *RTh* = 4 Ω

**P4.2 Substitution and Source Absorption Theorems**



**P4.2.1** Determine, according to the substitution theorem, (a) the independent voltage source, (b) the independent current source, and (c) the resistance that can replace the dependent current source in Figure P4.2.1 without affecting the rest of the circuit.



Ans. (a) 4 V; (b) 2 A; (c) 2 Ω.

**P4.2.2** Determine *VX* in Figure P4.2.2 by using the substitution theorem, where *NA* is an unspecified circuit that passes a current of 0.5 A.

Ans. 15 V.



**P4.2.3** Determine *VO* in Figure P4.2.3 by using the substitution theorem and by deriving NEC between nodes ‘ab’, where *NA* is an unspecified circuit having a voltage of 12.5 V across it.

Ans. -10/3 V.

**P4.2.4** Determine *IO* in Figure P4.2.4 by using the substitution theorem and by deriving TEC between nodes ‘ab’, where *NA* is an unspecified circuit that



passes a current of 12.5 A.

Ans. -10/3 A.

**P4.2.5** Determine *IO* in Figure P4.2.5 by using the substitution theorem and by deriving NEC between nodes ‘ab’, where *NA* is an unspecified circuit having a voltage of 15.5 V across it.



Ans. -22 A.

**P4.2.6** Determine *VO* in Figure P4.2.6 by using the substitution theorem and by deriving NEC between nodes ‘ab’, where *NA* is an unspecified circuit passing a current of 10 A.



Ans. 0.

**P4.2.7** Redo Example 4.4.1 assuming a current of 1.4 A in NA directed from left to right.

Ans. 105/281 = 0.37 A.



**P4.2.8** Determine *Rin* in Figure P4.2.4 by applying the source absorption theorem.

Ans. 40 Ω.



**P4.2.9** Determine *Rin* in Figure P4.2.9 by applying the source absorption theorem.

Ans. 100 Ω.

**P4.2.10** Determine *Rin* in Figure P4.2.6 by applying the source absorption theorem.



Ans. 1.25 Ω